PERMEABLE PLANAR COOLING THERMOELEMENT

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Abstract

Physical model of cooling thermoelement with a developed heat exchange system is described. Theory of calculation and computer methods to seek for optimal functions of the legs material inhomogeneity combined with a search for optimal parameters (electric current density, heat consumption) under which thermodynamic efficiency of power conversion will be maximum are discussed. Optimal inhomogeneity distribution for Bi_2Te_3 based material is given. Rational use of such power converters has been shown to increase coefficient of performance by 40-60%.

Introduction

There are thermoelements where heat exchange with the heat source and sink occurs not only on thermoelement junctions, but also in the bulk of the legs material [1-3]. Variants of realization of such models include permeable thermoelements wherein the legs materials along electric current flow employ channels (pores) for heat carrier pumping. Control over heat exchange conditions (heat carrier velocity, heat exchange intensity, etc.) combined with a distribution of physical effects in the legs material can affect the energy efficiency of power conversion.

Study of permeable thermoelements [3-5] has shown good prospects for their use, since it allows increasing coefficient of performance.

However, practical realization of such thermoelements is related to certain material research and technological difficulties, stimulating search for and study of the simplest variants of physical models of converters with internal heat exchange.

A variant of internal heat exchange realization includes planar thermoelements, wherein each leg consists of a certain number of plates, spaced from each other. Intervals between the plates form channels for pumping heat carrier (liquid or gas).

The purpose of this work is to study such thermoelements in order to determine their extreme energy characteristics of power conversion.

Problem formulation

Physical model of a planar permeable thermoelement working in thermoelectric cooling mode is shown in Fig.1. It comprises *n*- and *p*-type legs, each leg consisting of N_p segments (planes), spaced at the distance of h_k from each other. The segment width is h, and its thickness is h_p. Intervals between the segments form channels for pumping heat carrier (air or liquid) for cooling. Hot and cold thermoelement junctions at kept at constant values of T_h and T_c, respectively. Heat carrier is pumped in the direction from the hot to cold junctions.

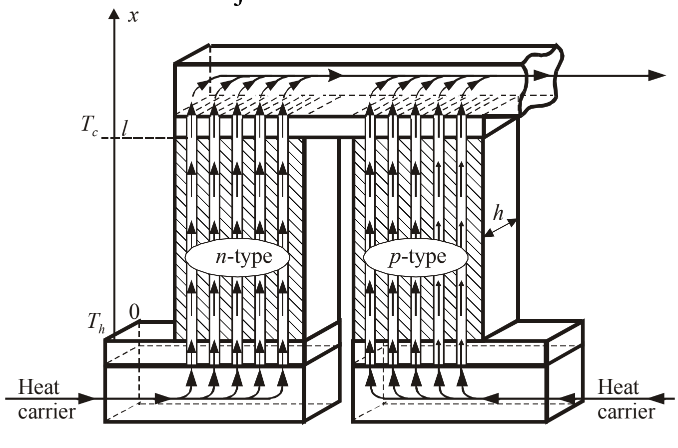

Fig. 1. Physical model of a planar cooling thermoelement.

Heat carrier temperature at thermoelement input is T_a. Heat exchange coefficient of heat carrier inside the channels of permeable junction thermoelement is α_T.

To find temperature distribution in thermoelement material, one should solve a differential equation

$$\begin{cases} \frac{dT}{dx} = -\frac{\alpha\ j\ T}{\kappa} - \frac{q}{\kappa}, \\ \frac{dq}{dx} = \frac{\alpha^2 j}{dx} T + \frac{\alpha\ j}{\kappa} q + i^2 \rho - \frac{\alpha_T \Pi_K N_K l^2}{(S - S_K) j}(T - t), \\ \frac{dt}{dx} = \frac{\alpha_T \Pi_K N_P l}{V c_P S_R}(T - t). \end{cases}_{n,p} \quad (1)$$

where t is heat carrier temperature at point x; T is leg temperature at point x; q is thermal flow; $q = \alpha(T,\xi) iT - \kappa(T,\xi)\frac{dT}{dx}$, j = il, $x = \frac{x}{l}$, α_T is heat exchange coefficient; i is electric current density ($i = \frac{I}{S - S_K}$); l is thermoelement height; $\Pi_K = 2 \cdot h$ is perimeter of a channel taking part in heat exchange; $S_k = h \cdot h_k \cdot N_p$ is cross-sectional area of all the channels; $S = h \cdot (h_k + h_p) \cdot N_p$ is cross-sectional area of the leg (leg material together with the channels). V is specific mass velocity of heat carrier in the channel ($V = v\rho_T$; v – velocity, ρ_T – heat carrier density); c_P – specific heat of heat carrier; $\alpha(T)$, $\kappa(T)$, $\rho(T)$ are thermoelectric coefficient, thermal conductivity and resistivity of material that are functions of temperature T. Note that parameters of thermoelectric medium α, κ, ρ are interdependent. The system of these relations assigns a certain area $G\xi$ of a change in inhomogeneity ξ. Elaborating on leg material, it is necessary to assign these relations, for example, in the form of theoretical or experimental dependences α, κ, ρ on T and define the area $G\xi$.

Let us consider the problem of maximum power efficiency of thermoelectric cooling under fixed heat source temperatures T_h and T_c.

The problem is reduced to search for maximum coefficient of performance

$$\varepsilon = \frac{Q_c}{Q_h - Q_c} \quad (2)$$

in case of differential relations (1) and boundary conditions

$$T_{n,p}(0) = T_h, \quad T_{n,p}(1) = T_c, \quad t_{n,p}(0) = T_s, \quad (3)$$

where T_h is junction hot surface temperature, T_c is junction cold surface temperature, T_s is initial temperature of heat carrier, Q_h, Q_c are thermal flows which thermoelement exchanges with external heat sources $Q_h = Q_n(0) + Q_p(0)$, $Q_c = Q_n(1) + Q_p(1) + Q_L$; here Q_L is heat input due to internal heat exchange from cooled heat carrier $Q_L = \sum_{n,p} V c_P S_R (t(0) - t(1))$.

Hereinafter instead of maximum ε it is convenient to consider functional minimum $\Im$:

$$\Im = \ln q(0) - \ln q(1), \quad (4)$$

where

$$q(0) = \frac{Q_h}{I} = q_n(0) + q_p(0),$$

$$q(1) = \frac{Q_c}{I} = q_n(1) + q_p(1) + \frac{Q_L}{j(S - S_K)} l,$$

here $q_n(1)$, $q_p(1)$, $q_n(0)$, $q_p(0)$ are the values of specific heat flows on the cold and hot junctions of thermoelement for n and p-type legs, found from solving the system of differential equations (1).

Optimization problem lies in the fact that from a variety of permissible controls $\xi \in G_\xi$ *to select* such concentration functions $\xi^{n,p}(x)$ and simultaneously *assign* such specific mass velocity of heat carrier in channels $V = V_O$ that in the case of *restrictions* (1)-(3) and *condition* for electric current density j

$$q_n(1) + q_p(1) = 0 \quad (5)$$

impart to *functional* $\Im$ *the least value*, the coefficient of performance ε in this case being maximum [6].

Method of solving formulated problem

To solve the problem, let us use mathematical theory of optimal control developed under the supervision of L.S. Pontryagin [7]. Let us formulate in brief the basic concepts of this theory.

Solution of optimal problems, based on the use of principle of the maximum, can be realized by numerical methods with elaboration of the respective computer programs.

The above formulated principle of the maximum is the basic result of theory of optimal processes. It can help to study various optimal control problems differing in the means of assigning functional (Lagrangian, Mayer, Boltz problems), restrictions, etc.

Let us elaborate on the formalism of mathematical theory of optimal control set forth before with regard to our problem.

Let us introduce the Hamiltonian function

$$H = \psi_1 f_1 + \psi_2 f_2 + \psi_3 f_3 \ , \qquad (6)$$

here $f_1, f_2,\ f_3$ are right-hand sides of equation system (1).

Functions $\psi(x)$ (pulses) must satisfy the system of equations:

$$\left\{\begin{aligned} &\frac{d\psi_1}{dx} = \frac{\alpha\ j}{\kappa} R_1 \psi_1 - \left(\frac{\alpha\ j}{\kappa} R_2 - \frac{\alpha_T \Pi_K N_K l^2}{(S-S_K)\,j}\right)\psi_2 - \frac{\alpha_T \Pi_K N_K l}{V c_P S_R}\psi_3,\\ &\frac{d\psi_2}{dx} = \frac{j}{\kappa}\psi_1 - \frac{\alpha\ j}{\kappa}\psi_2,\\ &\frac{dt}{dx} = -\frac{\alpha_T \Pi_K N_K l^2}{(S-S_K)\,j}\psi_2 + \frac{\alpha_T \Pi_K N_K l}{V c_P S_R}\psi_3. \end{aligned}\right._{n,p}$$

where

$$\left\{\begin{aligned} &R_1 = 1 + \frac{d\ln\alpha}{dT}T - \frac{d\ln\kappa}{dT}\left(T + \frac{q}{\alpha}\right)\\ &R_2 = R_1 + \frac{1}{Z_K}\frac{d\ln\sigma}{dT} + \frac{d\ln\kappa}{dT}\left(T + \frac{q}{\alpha}\right) \end{aligned}\right._{n,p}$$

canonically to conjugate system (4).

With boundary conditions (transversality conditions)

$$\psi_2^{n,p}(0) = \frac{1}{q_n(0) + q_p(0)},$$

$$\psi_2^{n,p}(1) = -\frac{(S-S_K)\,j}{lVc_P S_R\left(2t(0) - t_n(1) - t_p(1)\right)},$$

$$\psi_3^{n,p}(1) = -\frac{1}{2t(0) - t_n(1) - t_p(1)}.$$

Using this system of equations with regard to relations (1),(5) and numerical solution methods, program of computer design of optimal functions of thermoelectric material inhomogeneity $\xi(x)$ and optimal heat carrier velocity V was created with a view to achieve maximum energy efficiency of permeable planar cooling element.

Results of research on Bi_2Te_3 material

Let us give the results of computer design of optimal inhomogeneity of semiconductor thermoelectric material combined with optimal function of distribution of heat sources (sinks) for permeable planar cooling thermoelements. Approximated experimental temperature dependences of parameters α, κ, ρ of Bi_2Te_3 based material for *n*- and *p*-types for different impurity concentrations were used. Results of inhomogeneity optimization of semiconductor materials of *n*-, *p*-types and distribution of heat sources in thermoelement legs at temperature difference on its junctions 50 K, N_p=5шт., h_k=0.1 cm; h=1cm, l=1cm, h_p=0.1cm, T_h= 300 K, T_a= 300 K are shown in Fig. 2.

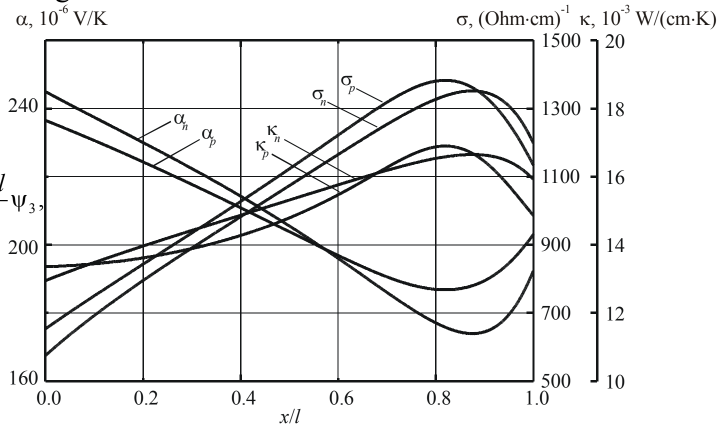

Fig. 2. Optimal distribution of material parametersalong height l of the legs of permeable planar thermoelement for temperature difference ΔT=40 K, T_h=300 K (x/l – dimensionless leg coordinate).

Results of search for optimal inhomogeneous or, as they are commonly referred to, functionally graded materials (FGM) of *n*- i *p*-type, agreed with optimal distribution of heat sources are shown in Fig.2 as a change in material parameters: thermoEMF α, electric conductivity *σ*, thermal conductivity *κ* (at 300K) along the legs height.

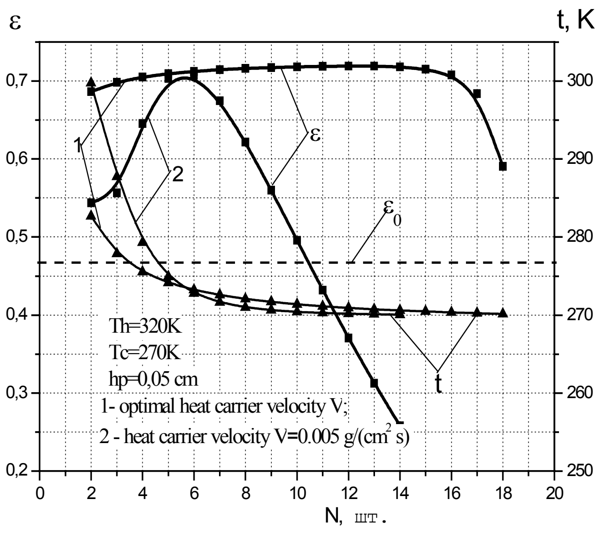

Fig.2. Dependece of coefficient of performance of permeable planar thermoelement ε of optimall parametres (dependence 1) and constant velocity for different number segments (planes) N.

The resultes of investigation demonstrat that there is rational number of segments *N* (fig.3), when thermoelement characteristics have the most favourable values. Similar data can be also obtained for other legs heights.

Comparison of the efficiency of operation of permeable and nonpermeable thermoelements in thermodynamic efficiency of cooling process is shown in Fig. 4. This figure shows the increase in coefficient of performance of permeable planar thermoelement of optimally homogeneous materials (dependence 1) and of functionally graded materials (dependence 2) with respect to its value for the bulk homogeneous thermoelment. Comparison shows that coefficient of performance of homogeneous permeable thermoelement is factor of 1.6÷2 higher than that of the nonpermeable one.

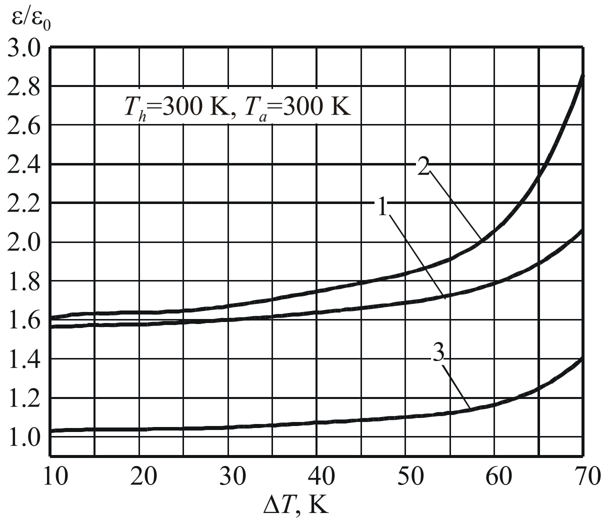

Fig. 4. Increase of coefficient of performance of permeable planar thermoelement ε of optimally homogeneous materials (dependence 1) and of functionally graded nmaterials (dependence 2) with respect to its value for nonpermeable homogeneous thermoelement ε_0 fordifferent temperature differences. Dependence 3 – additional increase of coefficient of performance of permeable planar FGM thermoelement with respect to its value for permeable homogeneous thermoelement.

The use of functionally graded thermoelectric materials for permeable thermoelements under boundary temperature differences (60÷70 K) allows additional increase of coefficient of performance by a factor of 1.2÷1.4 (dependence 3).

Thus, the efficiency of permeable thermoelements of materials with programmed inhomogeneity is 2÷2.8-fold higher compared to traditionally used homogeneous bulk thermoelements.

Conclusions

1. Theory of calculation and optimization of permeable planar thermoelements in cooling mode has been elaborated. Computer methods have been developed to seek for optimal functions of legs material inhomogeneity combined with a search for optimal parameters (electric current density, heat carrier consumption), when

thermodynamic efficiency of power conversion will be maximum.

2. Optimal distribution of inhomogeneities for *Bi-Te*-based legs material has been obtained, in which case under given thermal physical conditions the Peltier and Thomson bulk thermoelectric effects are best realized, giving maximum value to coefficient of performance.
3. Computer research on permeable planar thermoelements of *Bi-Te*-based materials has been made for various temperature operating conditions. It has been shown that rational use of such power converters allows increasing coefficient of performance by 40-60%.
4. The above results testify to good prospects of creating permeable planar thermoelements for thermoelectric coolers of increased efficiency.

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