Thermoelectric Device as an Attenuator of Density of Thermal Flux

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Abstract

Functioning of the thermoelectric cooling device leg that has a cross section, which is increased to the heat rejecting side discussed. It is proved that if isothermal surfaces are sufficiently flat then thermal processes can be described by the usual equation of heat balance with efficient values of thermoelectric parameters. The numeric calculation for legs of different forms made. It shown that the legs can attenuate the heat flux density by several times. The heat flux density can be order of magnitude of hundreds W/cm^2. The influence of contact resistance on maximal density of thermal flow discussed.

Introduction

The necessity to take off the thermal flux of very big density arises in modern electronics and optoelectronics quite often. Using metals with big thermal conductivity (Ag, Cu) for this purposes leads to big thermal losses.

The Peltier heat is characterized by the dependence only on the value of the current but not on its density. That is why it seems possible to use different Peltier devices for absorbing and dissipating the thermal flux of big density. Further, functioning of the thermoelectric leg capable of diminishing the density of thermal flux is studied.

Functioning the thermoelectric leg with alternating cross-section

Let us consider the thermoelectric leg with alternating cross-section $s(x)$, where x is the coordinate along the leg. Let the dependence of the cross section of the leg be given by the expression:

$$s(x) = s_0(1 + f(x)), \quad (1)$$

where $f(x)$ is an arbitrary function, that equals zero when $x=0$, and s_0 is the cross section of the leg on the cold side for which $x=0$. As for the other side of the leg with length L the function $f(L)>0$, i.e. cross section of the leg grows to the hot side. The ratio of the cross section on the hot side to that on the cold side is $\delta = \frac{s}{s_0} = 1 + f(L)$.

To find the temperature distribution along the leg it is necessary to solve 3D heat-transfer equation:

$$(\nabla, \kappa_0 \nabla T) + (\vec{j}, \vec{j})\rho_0 - T(\vec{j}, \nabla\alpha_0) = 0, \quad (2)$$

where κ_0 is thermal conductivity of the thermoelectric material, ρ_0 is its resistivity, α_0 is its Seebeck coefficient, and j is the current density. Further it is supposed that the thermoelectric parameters α_0, ρ_0, κ_0 do not depend on temperature. It is a simplifying assumption, but it does not change the essence of the problem. If in solution of the equation (2) isothermal surfaces are almost flat then it is convenient to proceed from 3D equation to 1D equation assuming that the leg has a constant cross section, and the thermal and electrical conductivities have the following dependence on coordinate x:

$$\sigma(x) = \sigma_0(1 + f(x)), \quad (3)$$

$$\kappa(x) = \kappa_0(1 + f(x)), \quad (4)$$

then 3D equation can be changed to 1D equation:

$$\frac{d}{dx}\left(\kappa(x)\frac{dT}{dx}\right) + j^2\rho(x) = 0, \quad (5)$$

that must be solved with the boundary conditions:

$$Q_o = a_0 I T_0 - \kappa_0(x) s_0 \left.\frac{dT}{dx}\right|_{x=0}, \quad (6)$$

$$Q = a_0 I T + \kappa_0(x) s_0 \left.\frac{dT}{dx}\right|_{x=L}, \quad (7)$$

where Q_0 is the thermal flux absorbed on the cold side of the leg (heat pumping capacity), Q is the thermal flux rejected from the hot side of the leg, T_0 is the temperature of the cold side, T is the temperature of the hot side, and I is the current. As the Seebeck coefficient in accordance with the accepted assumption does not depend on the temperature or on the coordinate, the term with $\nabla\alpha_0$ in equation (5) is not necessary. In order to receive the solution of (5) taking into account (3) and (4) it is convenient to use the method of efficient values proposed in [1,2]. According to this method the heat balance equation, which is received from (5) – (7), has the same form as in the simplest case when the thermoelectric parameters are constant, but the values of the thermoelectric parameters are substituted by their effective values. The effective value of thermal conductance according to [1,2] is the following:

$$\bar{k} = \frac{s_0 \kappa_0}{\int_0^L \frac{dx}{1 + f(x)}} = \frac{\bar{s}\kappa_0}{L}, \quad (8)$$

where

$$\bar{s} = \frac{s_0}{\frac{1}{L}\int_0^L \frac{dx}{1 + f(x)}}. \quad (9)$$

The efficient resistance for the cold side of the leg $R_{c.eff}$ has the following form:

$$R_{c.eff} = \frac{2\bar{k}}{L s_0}\int_0^L \frac{dy}{\sigma_0(1 + f(y))}\int_y^L \frac{dx}{\kappa_0(1 + f(x))} = \frac{L}{\bar{s}\sigma_0}, \quad (10)$$

This value of $R_{c.eff}$ is the same as the average resistance of the leg $\bar{R}$, and as the value of the efficient resistance for the hot side of the leg $R_{h.eff}$:

$$R_{c.eff} = R_{h.eff} = \bar{R}. \quad (11)$$

The heat balance equation for both sides of the leg is as follows:

$$\alpha_0 I T_0 - \frac{1}{2} I^2 \overline{R} - \overline{k} \Delta T = Q_0, \qquad (12)$$

$$\alpha_0 I T + \frac{1}{2} I^2 \overline{R} - \overline{k} \Delta T = Q, \qquad (13)$$

where $\Delta T = T - T_0$.

As the equations (12) – (13) have an ordinary form all the expressions describing the usual leg can be used for the abovementioned case substituting resistance and thermal conductance by the efficient values $\overline{R}$ and $\overline{k}$.

For example, when the thermoelectric leg is used for dissipation of thermal fluxes and the temperature difference between both sides of the leg equals zero: $\Delta T = 0$, the ratio of the densities of thermal fluxes on the sides of the leg v_0 equals

$$v_0 = \frac{Q_0 \delta}{Q}. \qquad (14)$$

The maximum of Q_0 is obtained when the current equals

$$I_0 = \frac{\alpha T}{\overline{R}} = \frac{\alpha T \sigma_0 \overline{s}}{L}. \qquad (15)$$

The value of v_0 for this current is

$$v_0 = \frac{\delta}{3}. \qquad (16)$$

The attenuation of the heat flux density $v_0 > 1$ is attained when

$$\delta > 3. \qquad (17)$$

The attenuated flux density q_0 in this case is

$$q_0 = \frac{1}{2} \frac{\alpha^2 \sigma_0 T^2}{L} \frac{\overline{s}}{s_0}. \qquad (18)$$

If $\Delta T > 0$ then the ratio of the densities of the thermal flux is

$$v = \frac{\delta}{1 + \frac{1}{\varepsilon}}, \qquad (19)$$

where ε is the coefficient of performance (COP), which at maximum COP equals

$$\varepsilon = \frac{T_0}{\Delta T} \frac{M - \frac{T}{T_0}}{(M+1)}, \qquad (20)$$

where

$$M = \sqrt{1 + Z \frac{T_0 + T}{2}} \qquad (21)$$

and

$$Z = \frac{\alpha_0^2}{\overline{R}\overline{k}} = \frac{\alpha_0^2 \sigma_0}{\kappa_0}. \qquad (22)$$

If the values of δ satisfy the inequality

$$\delta > 1 + \frac{1}{\varepsilon}, \qquad (23)$$

then the density of the thermal flux can be diminished and the leg with the alternating cross section is an effective attenuator of the heat flux density.

In this case the value of the optimal current for maximum COP equals

$$I_\varepsilon = \frac{\alpha \Delta T}{\overline{R}(M-1)}, \qquad (24)$$

and the heat pumping capacity of the leg equals

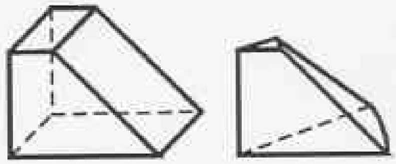
$$Q_0 = I_\varepsilon^2 \overline{R} \varepsilon M. \qquad (25)$$

The expression for the density of the thermal flux that can be attenuated at maximal COP received from (24) and (25) is the following:

$$q_0 = \frac{\alpha^2 (\Delta T)^2 \varepsilon M \sigma \overline{s}}{L(M-1)^2 s_0}. \qquad (26)$$

Implementation of the method

As an example, the functioning of the legs with the form as in fig.1 is discussed.



a) b)

Fig. 1. The form of the leg: a) a prism, b) a blunt-nosed cone.

For fig. 1 a)

$$f = \frac{\delta - 1}{L} x, \qquad (27)$$

and

$$\frac{\overline{s}}{s_0} = \frac{\delta - 1}{\ln \delta}. \qquad (28)$$

For fig.1 b)

$$f(x) = \left(1 + \frac{(\sqrt{\delta} - 1)x}{L}\right)^2 - 1, \qquad (29)$$

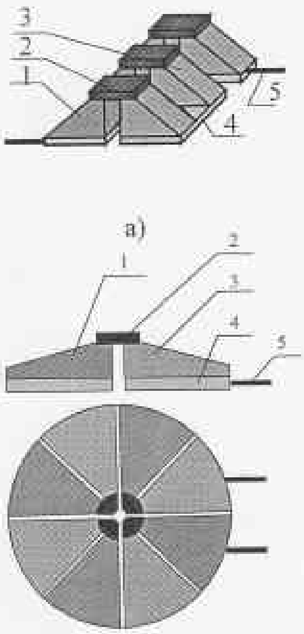
and

$$\frac{\overline{s}}{s_0} = \sqrt{\delta}. \qquad (30)$$

The density of the thermal fluxes that can be attenuated with the help of the legs fig.1 a) and b) can be calculated. First of all the case of $\Delta T = 0$ is examined. Let the thermoelectric materials have the usual properties $\alpha_0 = 200$ μV/K, $\sigma_0 = 1000$ Ω^{-1}cm^{-1}, L = 1mm, the temperature on both sides equals $T = T_0 = 350$ K, and $\delta = 10$. Then, for the leg represented on fig. 1 a) $q_0 = 96$ W/cm^2, and for the leg on fig 1 b) $q_0 = 77$ W/cm^2. For comparison it can be noted that the critical density of the thermal flux for nucleate boiling of water is 90 W/cm^2. The coefficient of diminishing the density of the thermal flux is $v = 3$. As the maximal attenuated density of the thermal flux is inversely proportional to the length of the leg L (17) there can be a considerable increase in the density of the thermal flux by diminishing the length of the leg. Besides, the values of δ exceeding 10 can be easily received for the leg on fig 1 b).

When $\Delta T > 0$ let $Z = 2.7\ 10^{-3}$ K^{-1} and $T_0 = 350$ K. It is clear that it is unlikely to function under condition $\varepsilon < 1$, therefore let $\varepsilon = 1$. The corresponding temperature difference is $\Delta T = 41$K. The density of the thermal flux that can be attenuated in this case is considerably less: for the leg fig. 1 a) $q_0 = 9.4$ W/cm^2, and for the leg fig 1 b) $q_0 = 7.6$ W/cm^2.

The legs on fig. 1 must be connected so as to create a thermoelectric battery. The possible method of connecting of the legs is given on fig. 2.



b)

Fig. 2. Possible method of creating a battery from the legs fig. 1. Indexes a) and b) on this figure correspond to indexes a) and b) fig.1. 1 – n-type leg , 2 – a connector on the cold side of the legs, 3 – p-type leg, 4 – a connector on the hot side of the legs, 5 – the leads.

These forms of the thermoelectric battery are convenient for rejecting thermal fluxes both from longitudinal and point heat sources.

Conclusions

The proposed method of diminishing the density of the thermal flux is based on assumptions, which are approximate, therefore in reality the results are expected to be worse than in the calculation above. Experimental examinations are to answer the questions of true value of the attenuation of heat flux density.

The proposed method can be used for rejecting the heat from the elements of electronic chips.

References

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